

# Subpixel Translation of MEMS Measured by Discrete Fourier Transform Analysis of CCD Images

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# Outline

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## Working principle

→ Discrete Fourier Transform applied to image analysis

## Computing software

→ MATLAB Graphical User Interface (GUI)

## Experimental results

- Static characterization of MEMS
- Dynamic characterization of MEMS

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2\sin^2 \theta \\ &= 2\cos^2 \theta - 1 \end{aligned}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow u'(x) = \frac{4}{1 - 4x^2 - 3}$$

$$\Rightarrow u' = \frac{4}{\sqrt{2x^2 - 3x + 1}}$$

$$8(2x^2 - 3x + 1)^{\frac{1}{2}} = \frac{4(2x - 3)}{\sqrt{2x^2 - 3x + 1}}$$

$$= \frac{4(2x - 3)}{\sqrt{2x^2 - 3x + 1}}$$

$$= \frac{16x^2 - 24x + 8 - 16x^2 + 24x - 8}{\sqrt{2x^2 - 3x + 1}}$$

$$= \frac{0}{\sqrt{2x^2 - 3x + 1}}$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1} \left( \frac{x}{a} \right) + C$$

$$\begin{aligned} \sin^2 ax &= \frac{1 - \cos 2ax}{2} \\ \cos^2 ax &= \frac{1 + \cos 2ax}{2} \\ \sin^2 ax &= (1 - \cos^2 ax) \sin ax \\ \cos^2 ax &= (1 - \sin^2 ax) \cos ax \end{aligned}$$

$$1 + \tan^2 ax = \sec^2 ax$$

$$\frac{d}{dx} \tan kx = k \sec^2 kx$$

$$\int \sec^2 ax dx = \tan ax + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \csc^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

## Working principle

- Discrete Fourier Transform applied to image analysis

# Working principle | Motivation

## Need for precise in-plane MEMS characterization

- Process dependent properties
- Device calibration

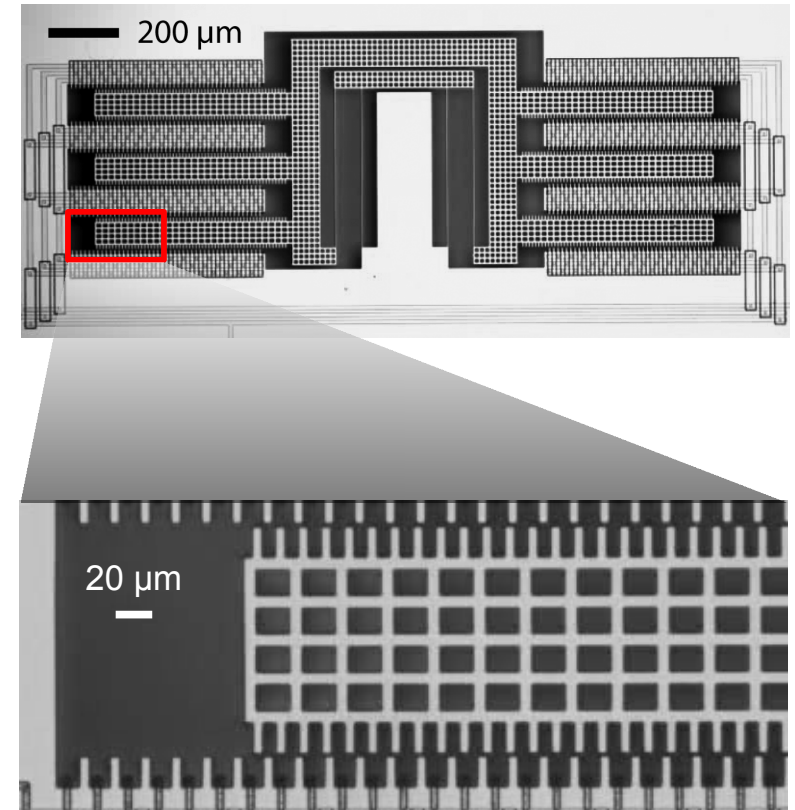


## Existing solutions

- Solid-state integrated sensor
- Optical methods (*e.g.* stroboscopy, blurred imaging, high speed camera)



We propose a straightforward optical method based on DFT image analysis.

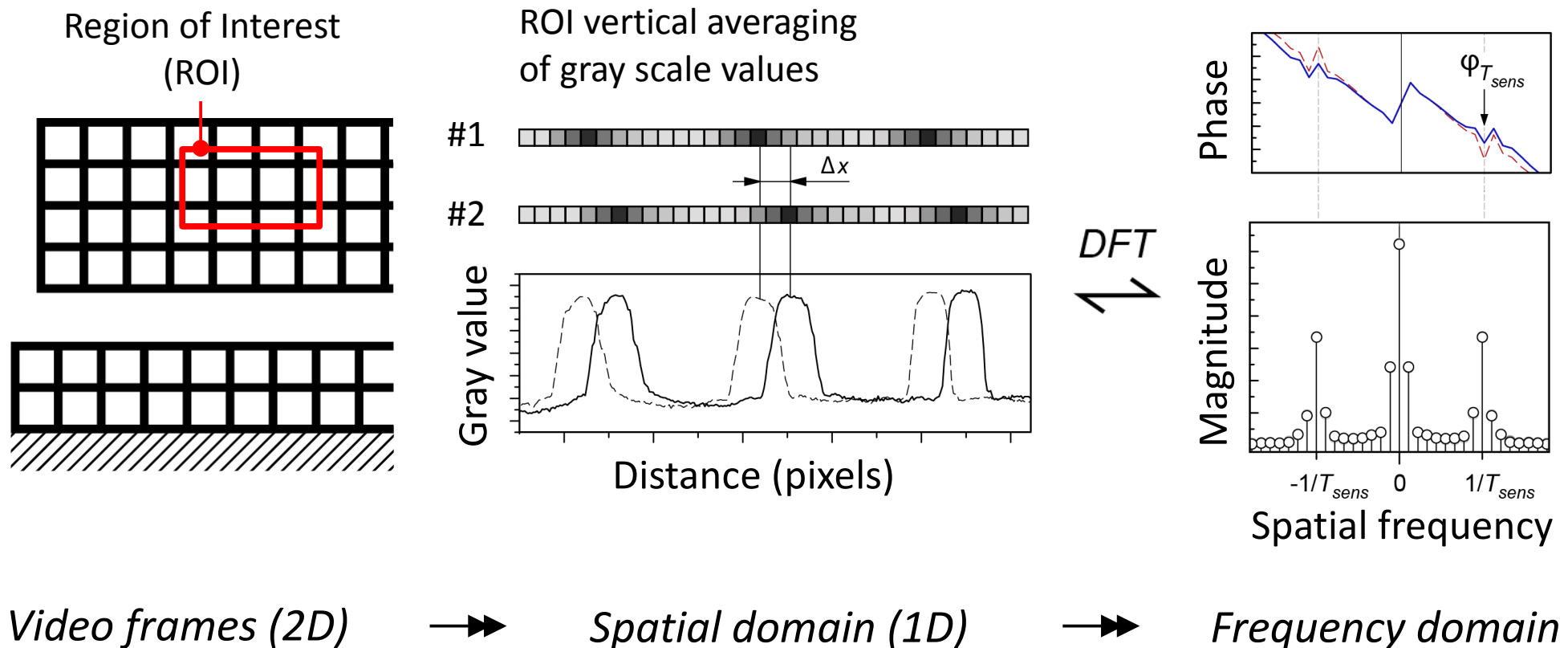


E. Sarajlic *et al.*, “Electrostatic 3-phase linear stepper motor fabricated by vertical trench isolation technology,” *J. Micromech. Microeng.* **19** (7), No. 074001, 2009

# Working principle | Discrete Fourier Transform

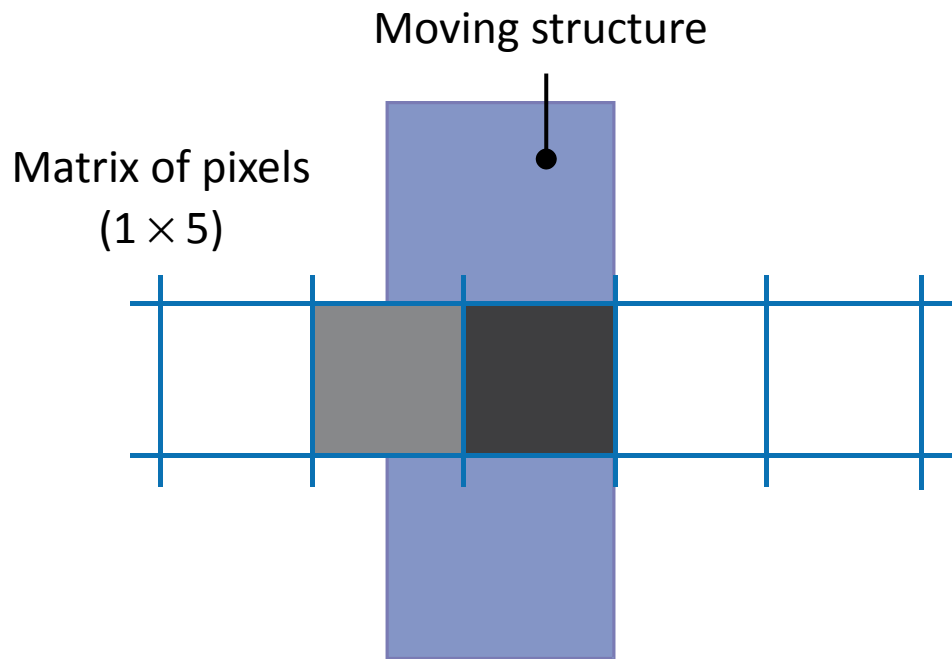
## Discrete Fourier Transform (DFT) analysis of moving periodic patterns

Algorithm based on the phase-shift method.



C. Yamahata, E. Sarajlic, G.J.M. Krijnen, and M.A.M Gijs,  
"Subnanometer Translation of Microelectromechanical Systems Measured by Discrete Fourier Analysis of CCD Images,"  
*J. Microelectromech. Syst.* **19** (5), pp. 1273-1275, 2010.

# Working principle | How subpixel resolution is possible?

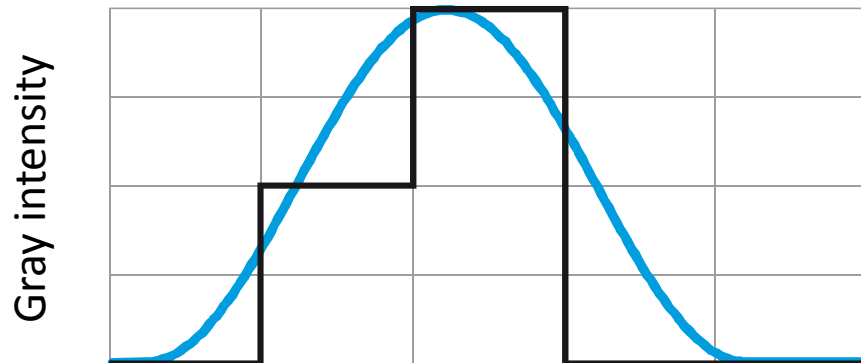


Subpixel resolution can simply be obtained by gray scale level interpolation.

Enhancement:

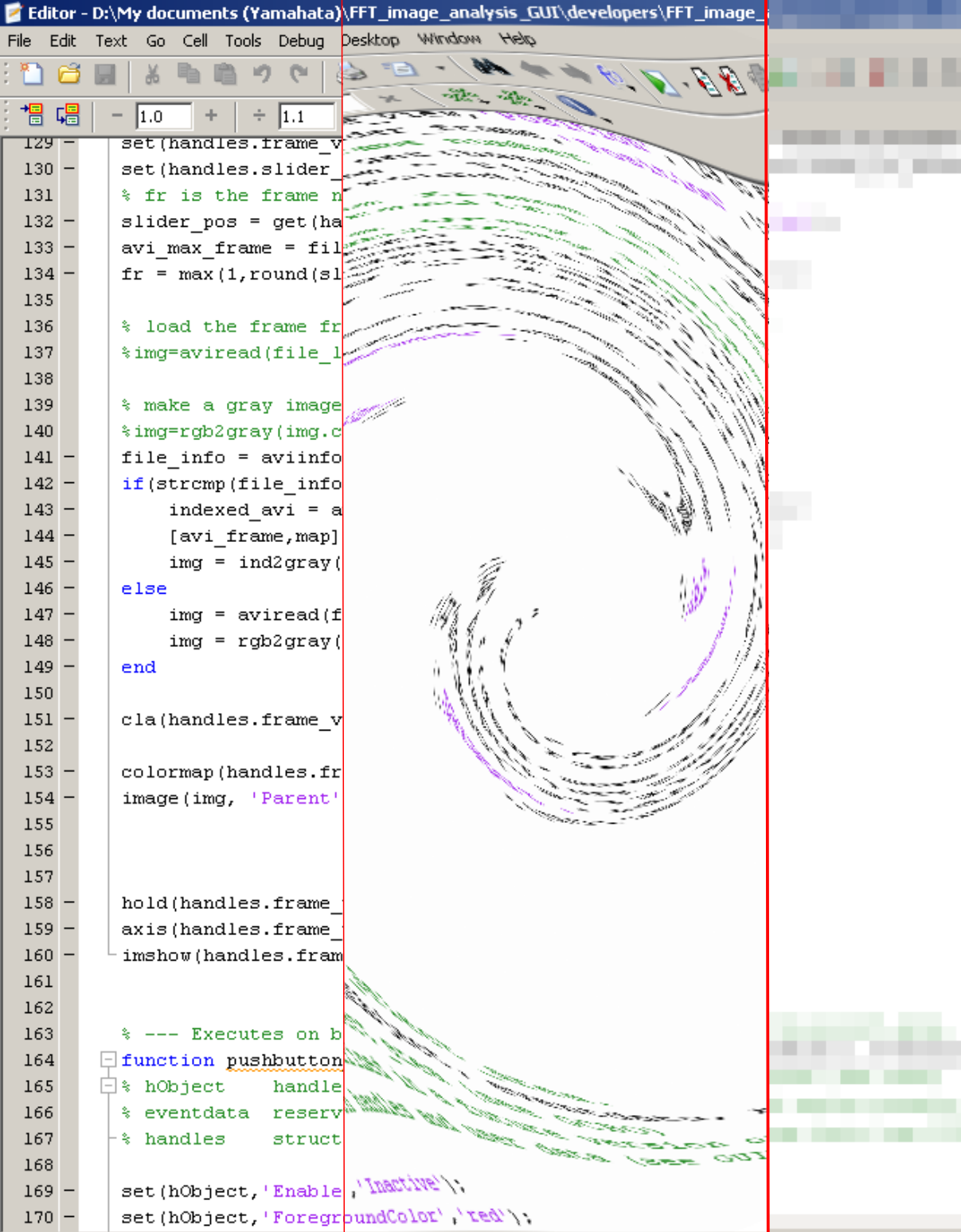
- Periodic patterns
- Large matrix of pixels

For further mathematical details, see reference below.



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C. Yamahata, E. Sarajlic, G.J.M. Krijnen, and M.A.M Gijs, "Subnanometer Translation of Microelectromechanical Systems Measured by Discrete Fourier Analysis of CCD Images," *J. Microelectromech. Syst.* **19** (5), pp. 1273-1275, 2010.



## Computing software

→ **MATLAB** Graphical User Interface (GUI)

# Computing software | MATLAB GUI

## Step 1: Video selection

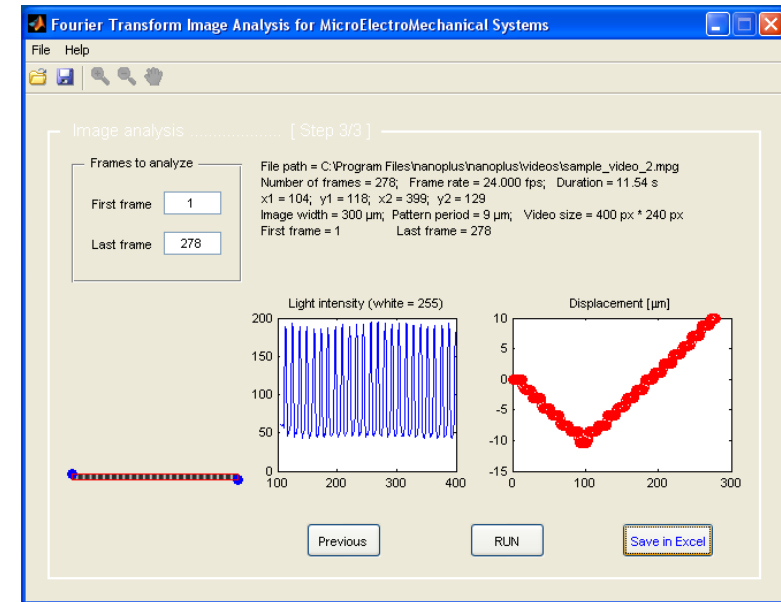
- AVI file (compressed or uncompressed)
- Any MATLAB compatible codec (e.g. mpeg, wmv)

## Step 2: Area selection

- Select the **region of interest (ROI)**
- Set the parameters:
  - **scale** (video width)
  - **spatial period** of the structure

## Step 3: Frame by frame analysis

→ Data can be **saved in Excel**



Demonstration video

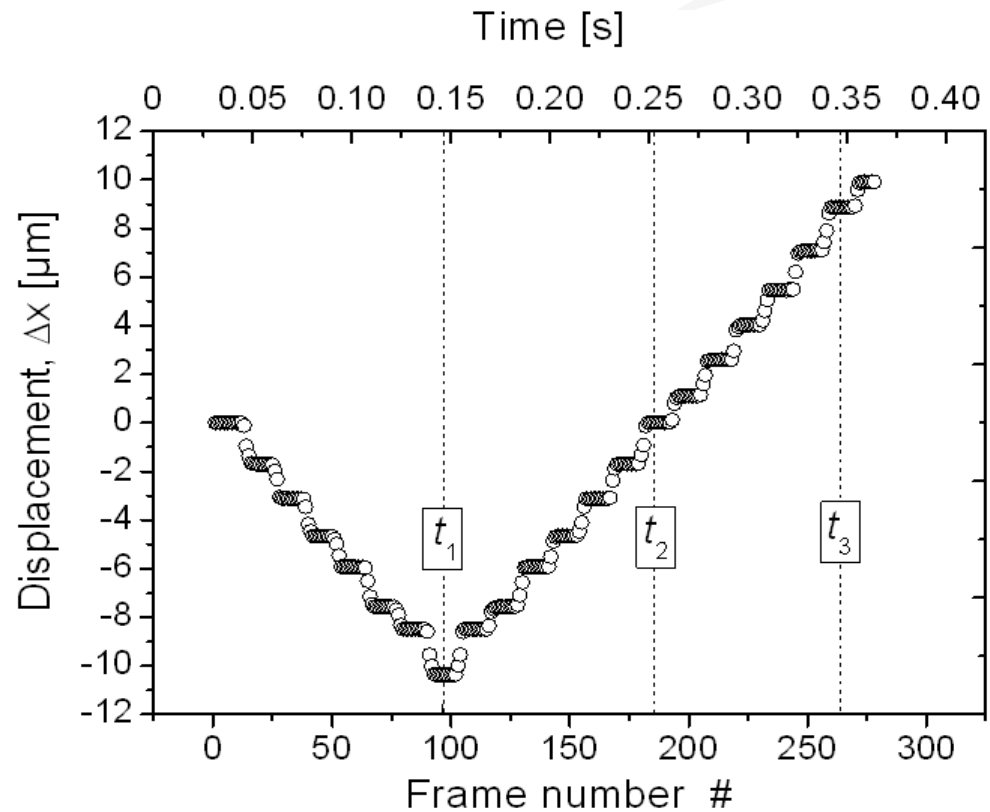
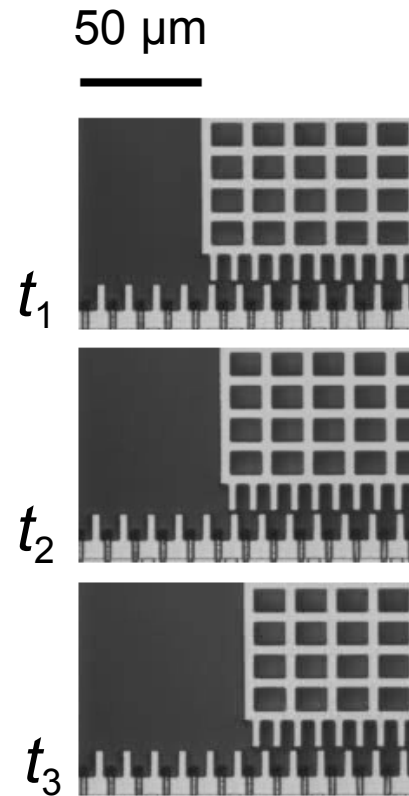


Software freely available at

<http://lmis2.epfl.ch/nanoplus/>

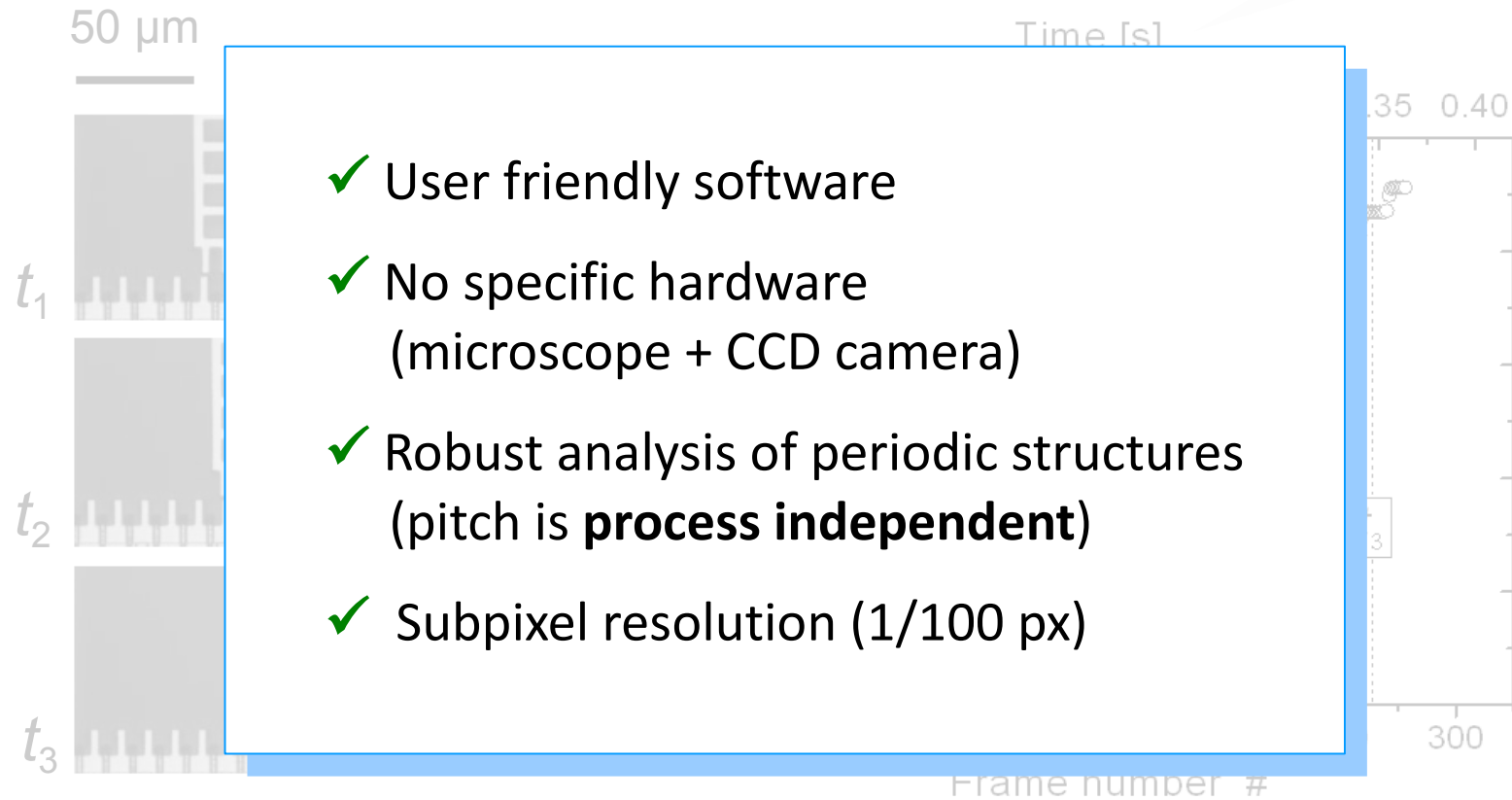


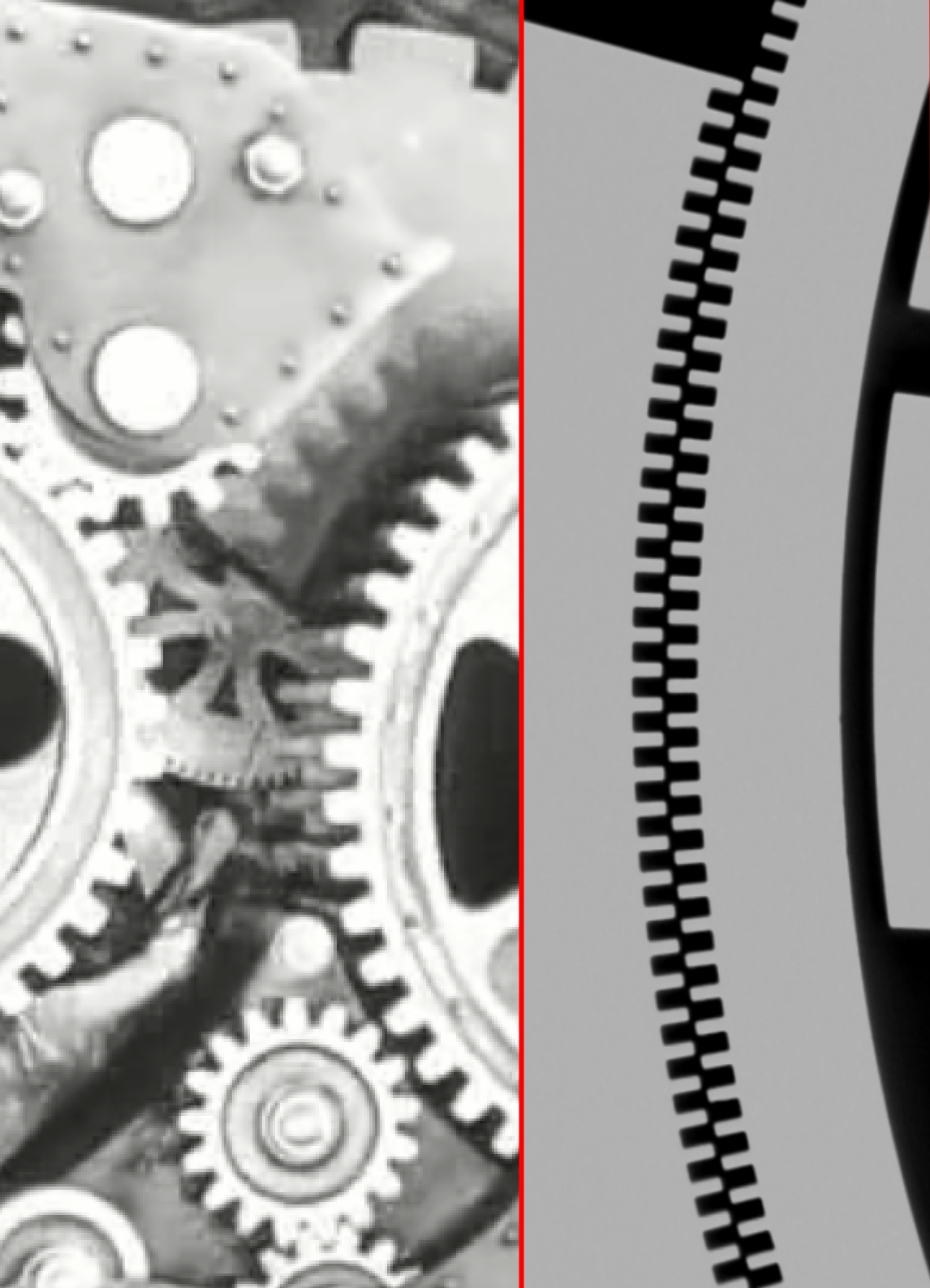
# Computing software I **MATLAB GUI**



$$\text{Time} = \frac{\text{Frame \#}}{\text{Frame rate}}$$

# Computing software | MATLAB GUI



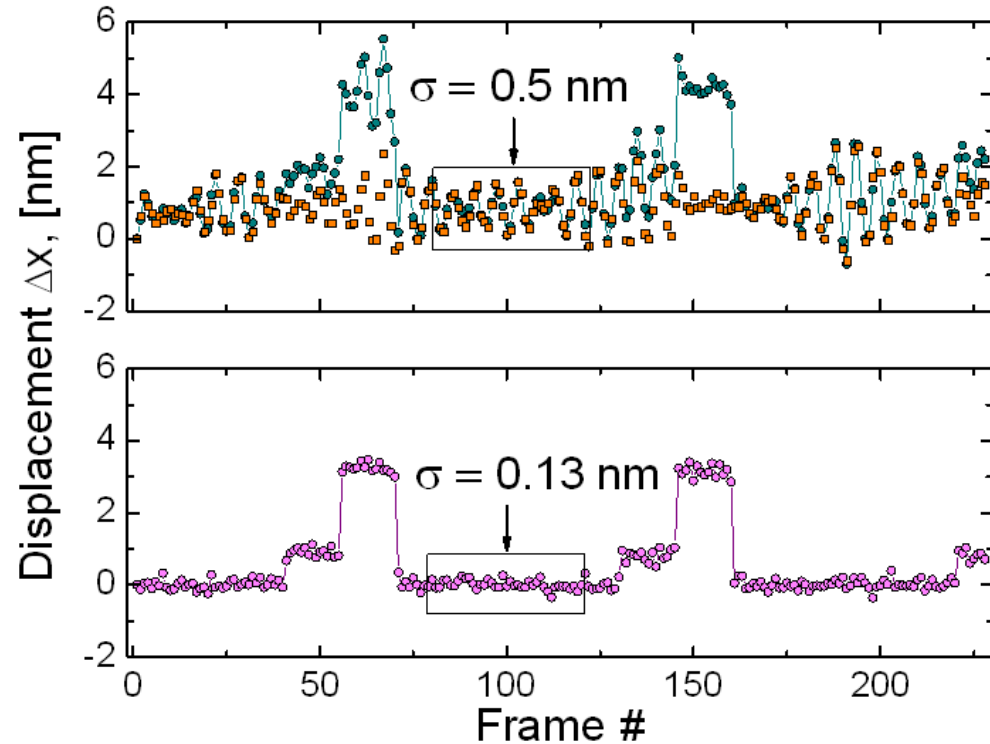
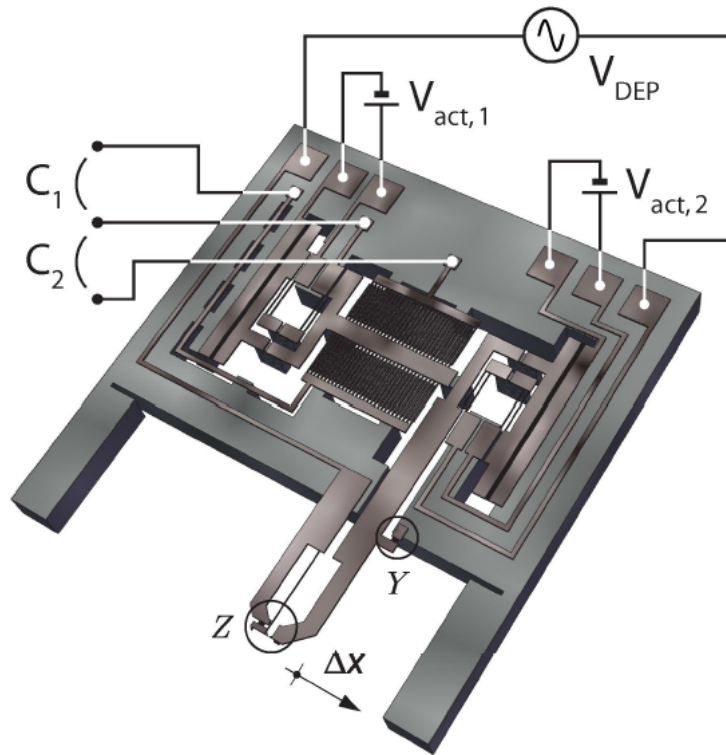


## Static measurements

- Subpixel resolution
- Effect of vibrations

# Static Measurements | Application to MEMS

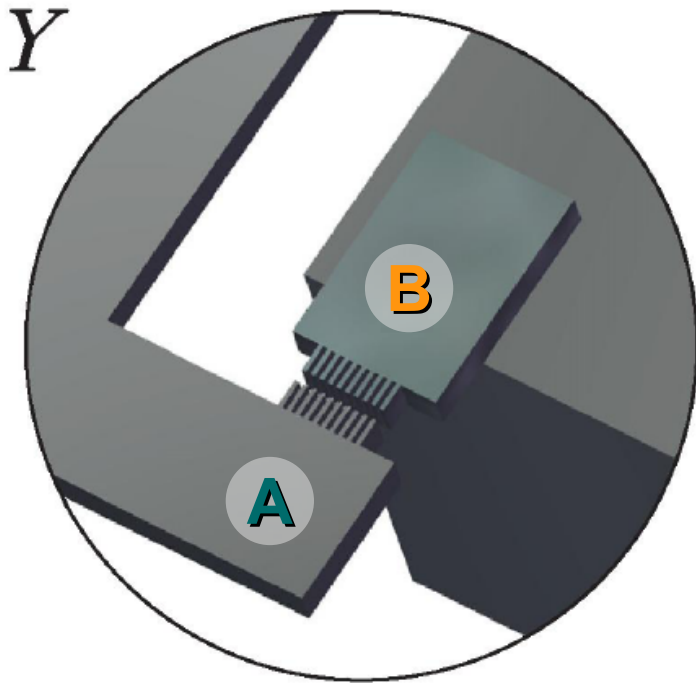
## Case 1: Subpixel resolution by relative displacement analysis



C. Yamahata, E. Sarajlic, L. Jalabert, M. Kumemura, D. Collard and H. Fujita,  
"Mechanical Characterization of Biomolecules in Liquid Using Silicon Tweezers with Subnanonewton Resolution,"  
*Proc. MEMS 2009*, pp. 607-610, Sorrento, Italy, 2009.

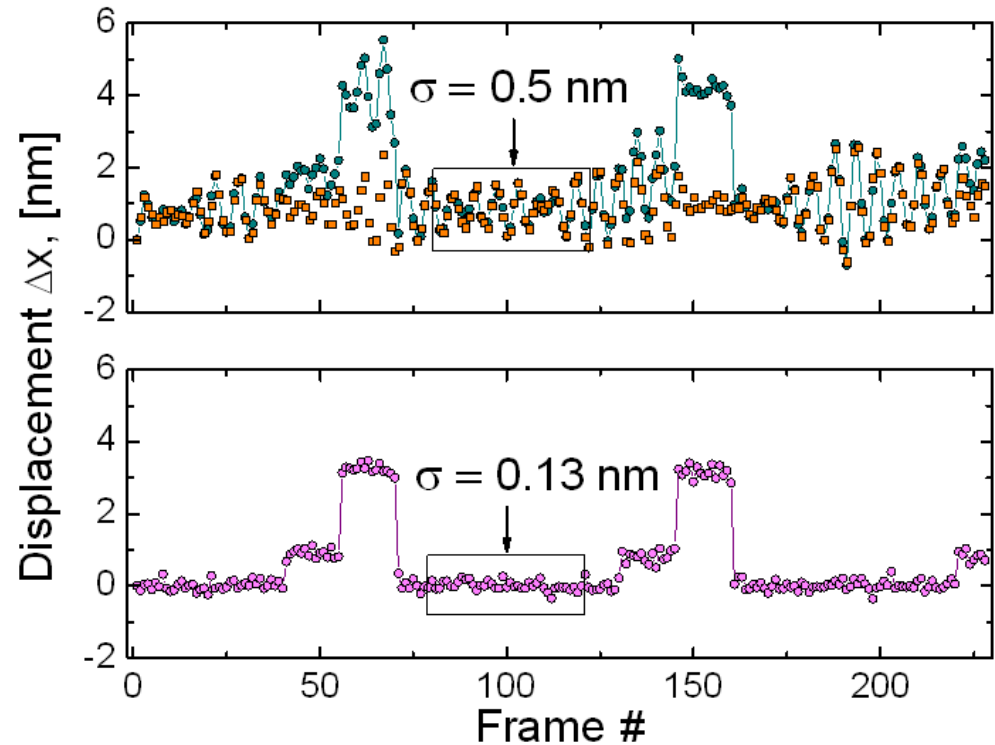
# Static Measurements | Application to MEMS

## Case 1: Subpixel resolution by relative displacement analysis



- A** Moving structure
- B** Reference patterns

● = **A** - **B**      Relative displacement

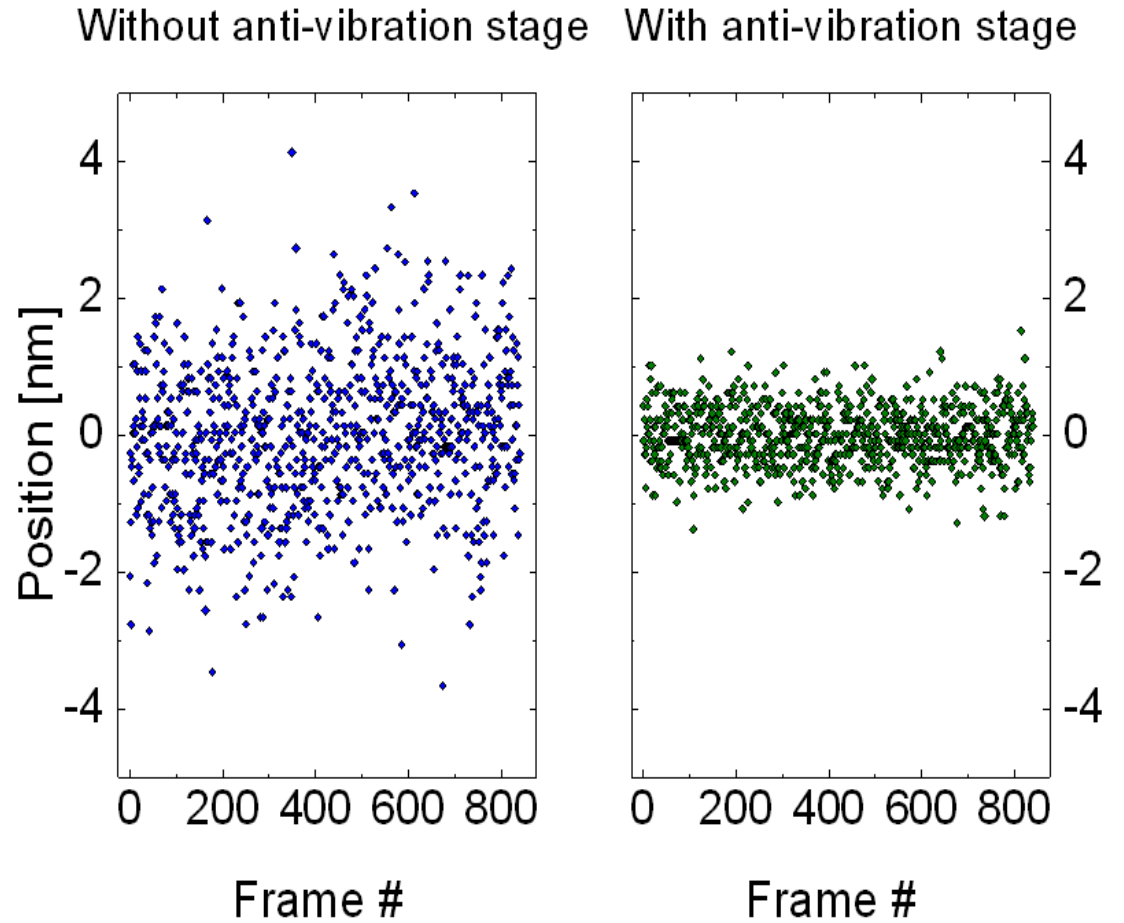
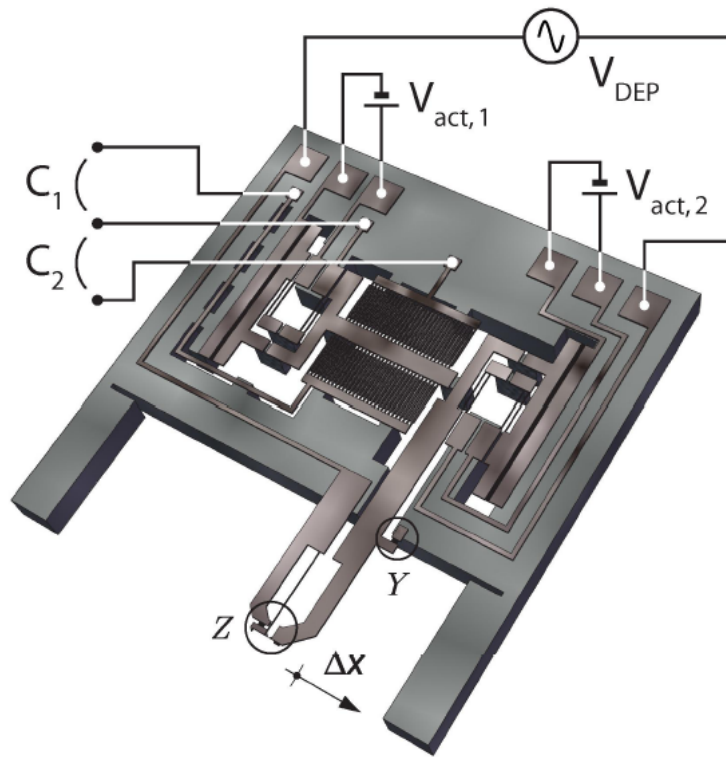


Pixel  $75 \times 75 \text{ nm} \Rightarrow 1/100 \text{ px resolution}$

C. Yamahata, E. Sarajlic, L. Jalabert, M. Kumemura, D. Collard and H. Fujita,  
"Mechanical Characterization of Biomolecules in Liquid Using Silicon Tweezers with Subnanonewton Resolution,"  
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# Static Measurements | Application to MEMS

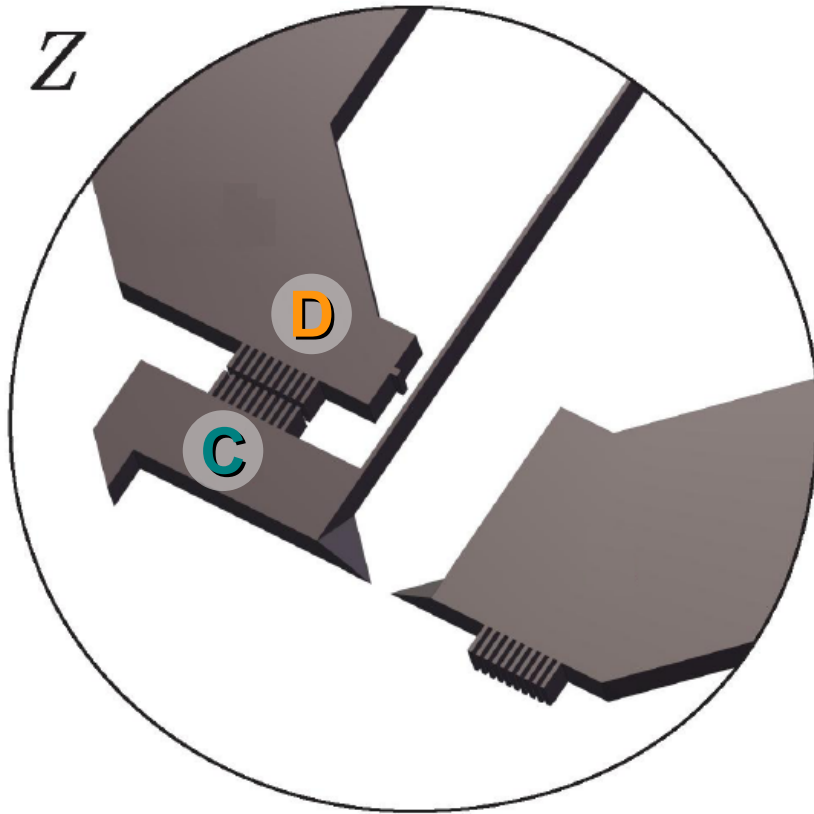
## Case 2: Effect of vibrations



# Static Measurements | Application to MEMS

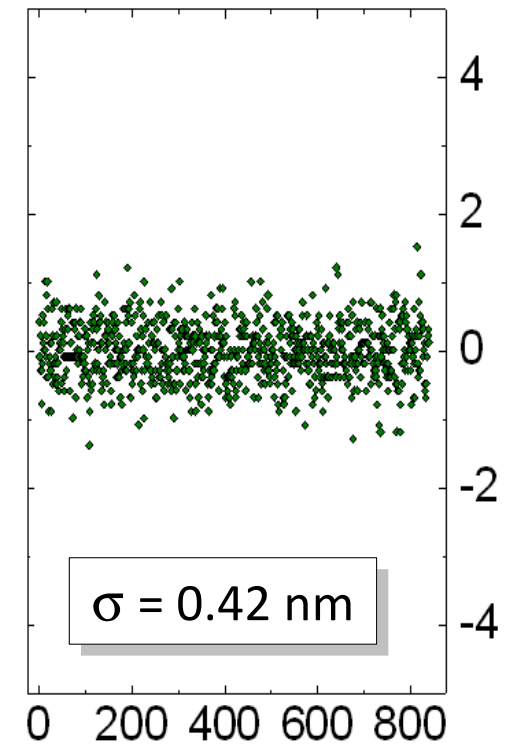
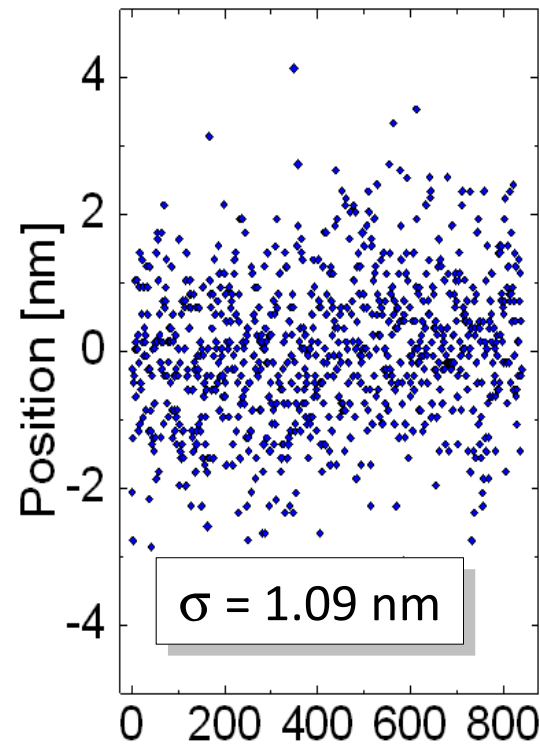
## Case 2: Effect of vibrations

●/● = C - D Relative displacement



- C Moving structure
- D Reference patterns

Without anti-vibration stage    With anti-vibration stage





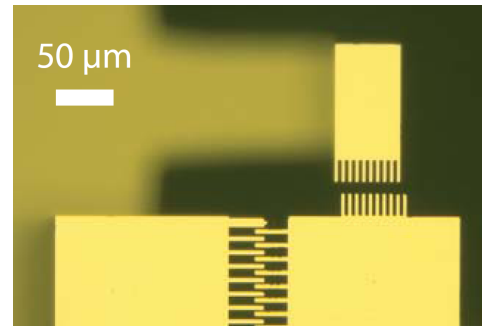
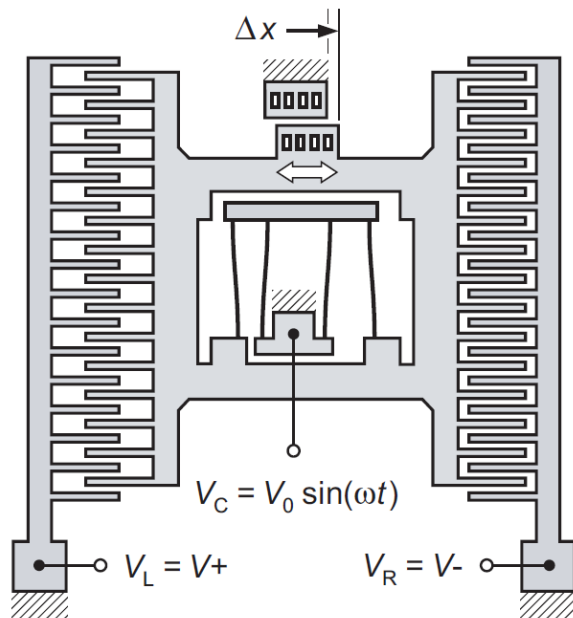
## Dynamic measurements

- Aliasing effect
- Temporally aliased video microscopy



# Dynamic Measurements | Current researches

## Case 3: Aliasing effect



Video frame rate:

$$f_s = 14.9763 \text{ Hz}$$

$$[f_s \approx 15 \text{ fps (tech. spec.)}]$$

Oscillation frequency:

$$f_M = \omega/2\pi = 2007 \text{ Hz}$$

Aliased frequency:

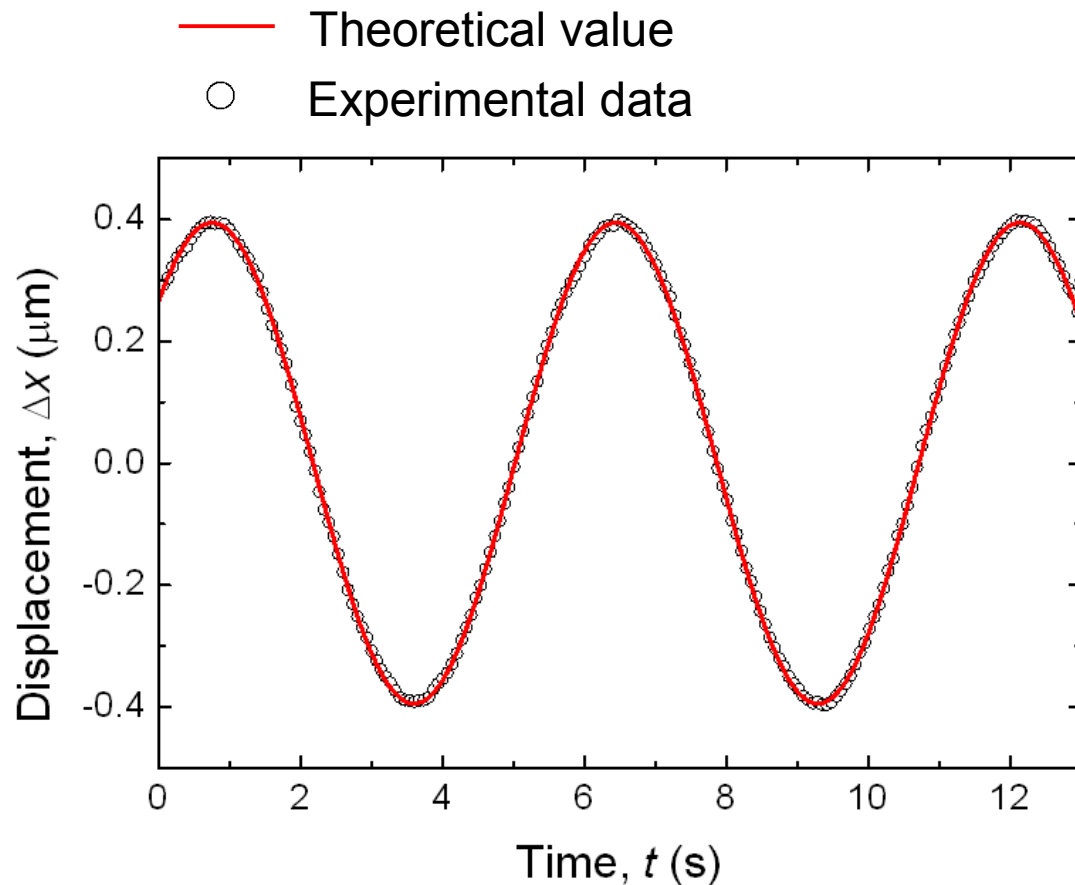
$$f_{\text{alias}} = |f_M - N \times f_s| = 0.1758 \text{ Hz}$$

with  $N = 134$

➡ Shutter set to 1/5000 s

# Dynamic Measurements | Current researches

## Case 3: Aliasing effect



Video frame rate:

$$f_s = 14.9763 \text{ Hz}$$

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Aliased frequency:

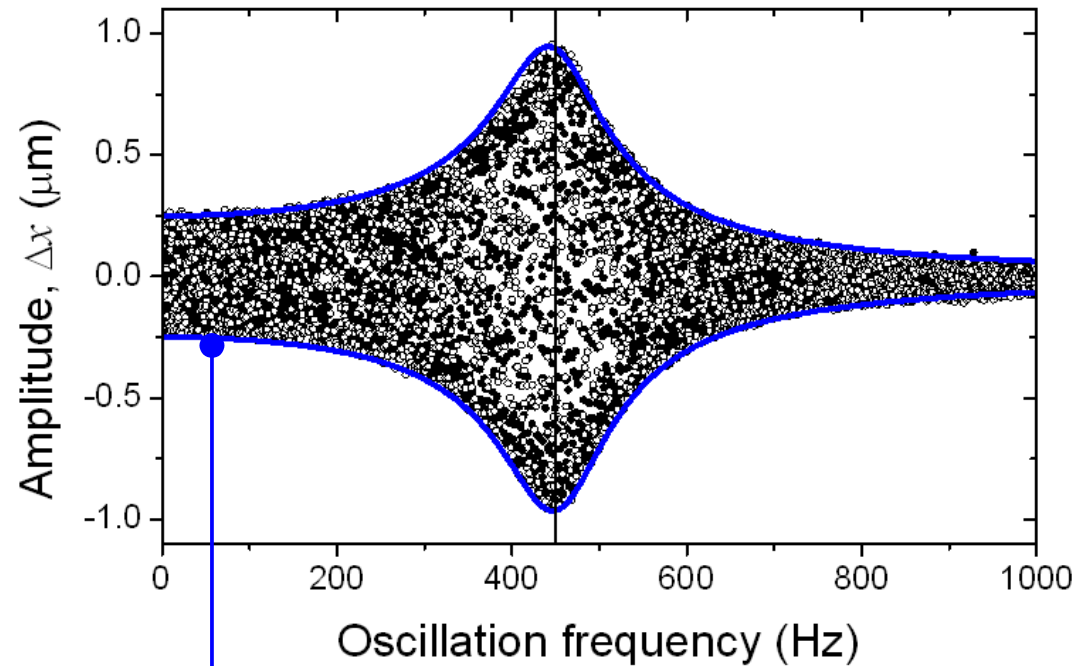
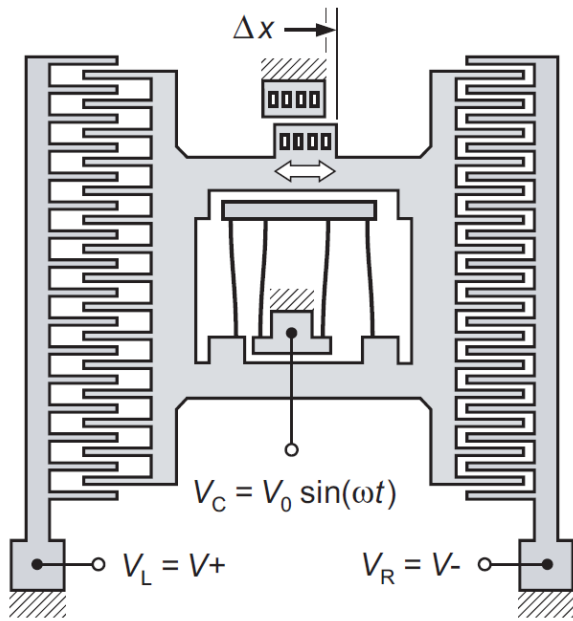
$$f_{\text{alias}} = |f_M - N \times f_s| = 0.1758 \text{ Hz}$$

with  $N = 134$

➡ Shutter set to 1/5000 s

# Dynamic Measurements | Current researches (unpublished)

## Case 4: Temporally aliased video microscopy



Damped harmonic oscillator  
(envelope computation)

Video frame rates:

$$f_s = 15 \text{ fps } (\circ) / 28 \text{ fps } (\bullet)$$

Frequency sweep:

$$f_M = 1 \text{ Hz} - 1 \text{ kHz}$$



Conclusion

# Conclusion

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Advantages of the proposed method:

- ✓ Subpixel resolution (1/100 px)
- ✓ In-situ measurement reference:  
Pattern period is process independent
- ✓ No specific hardware  
(microscope + CCD camera)
- ✓ User friendly software

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Applications in the field of MEMS:

- Calibration
- Strain–stress measurements at the  $\mu\text{m}$  scale
- Experimental modal analysis



Software freely available at <http://lmis2.epfl.ch/nanoplus/>



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SWISS NATIONAL SCIENCE FOUNDATION

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